Zeichne das Viereck $ABCD$ in ein Koordinatensystem, bestimme die Länge und Steigung der Strecken $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ und $\overline{AD}$, gib die Koordinaten der Mittelpunkte M_{AB}, M_{BC}, M_{CD} und M_{AD} an und berechne die Innenwinkel α, β, γ und δ.

a) $A(1|1), B(4|1), C(5|3), D(2|3)$

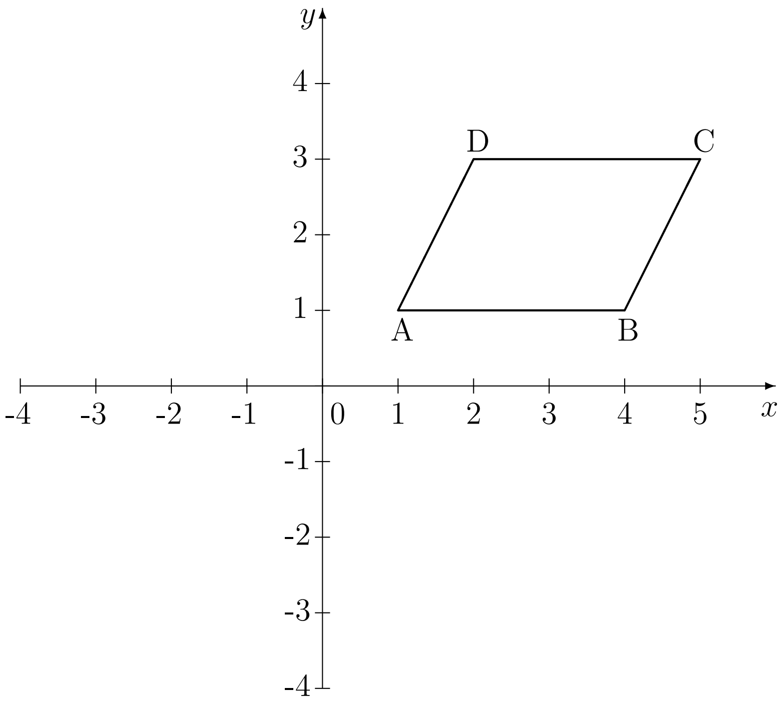
b) $A(1|-2), B(2|0), C(5|3), D(3|4)$

c) $A(1|1), B(5|1), C(6|2), D(3|4)$

d) $A(-3|-1), B(2|-3), C(3|3), D(-2|1)$

Lösung:

a)



Berechnung der Kantenlängen

$$\boxed{\overline{PQ} = \sqrt{\left(x_Q - x_P\right)^2 + \left(y_Q - y_P\right)^2}}$$

$\overline{AB} = 4 - 1 = 3$ (hier geht's auch ohne Pythagoras!)
$\overline{BC} = \sqrt{1^2 + 2^2} = \sqrt{5} \approx 2,24$
$\overline{CD} = 5 - 2 = 3$
$\overline{AD} = \sqrt{1^2 + 2^2} = \sqrt{5} \approx 2,24$

Berechnung der Kantensteigungen

$$\boxed{m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}}$$

$m_{AB} = \frac{1-1}{4-1} = 0$ $m_{BC} = \frac{3-1}{5-4} = 2$ $m_{CD} = \frac{3-3}{5-2} = 0$ $m_{AD} = \frac{3-1}{2-1} = 2$

Berechnung der Streckenmittelpunkte

$$\boxed{M_{PQ}\left(\frac{x_Q+x_P}{2}\,\middle|\,\frac{y_Q+y_P}{2}\right)}$$

$M_{AB}\left(\frac{4+1}{2}\,\middle|\,\frac{1+1}{2}\right) \equiv M_{AB}\,(2,5\;|\;1)$

$M_{BC}\left(\frac{5+4}{2}\,\middle|\,\frac{3+1}{2}\right) \equiv M_{BC}\,(4,5\;|\;2)$

$M_{CD}\left(\frac{5+2}{2}\,\middle|\,\frac{3+3}{2}\right) \equiv M_{CD}\,(3,5\;|\;3)$

$M_{AD}\left(\frac{2+1}{2}\,\middle|\,\frac{3+1}{2}\right) \equiv M_{AD}\,(1,5\;|\;2)$

Berechnung der Innenwinkel

$$\boxed{\varphi = \arctan(m_2) - \arctan(m_1)}$$

oder

$$\boxed{\varphi = 180^\circ - \big(\arctan(m_2) - \arctan(m_1)\big)}$$

oder

$$\boxed{\varphi = 180^\circ + \big(\arctan(m_2) - \arctan(m_1)\big)}$$

$\alpha = \arctan(m_{AD}) - \arctan(m_{AB}) \approx 63,43^\circ - 0 = 63,43^\circ$

$\beta = 180^\circ - \big(\arctan(m_{BC}) - \arctan(m_{AB})\big) \approx 180^\circ - (63,43^\circ - 0) = 116,57^\circ$

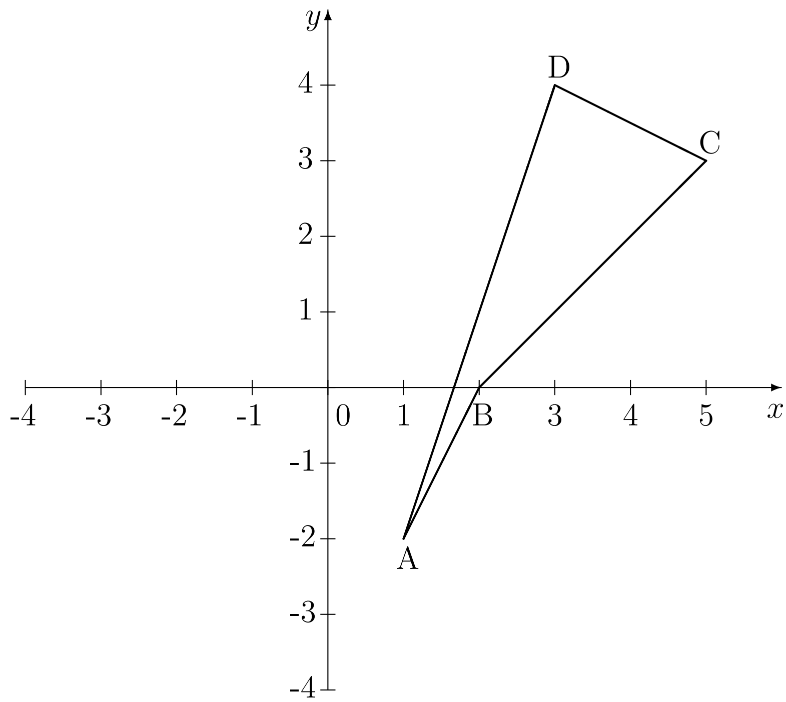
$\gamma = \arctan(m_{BC}) - \arctan(m_{CD}) \approx 63,43^\circ - 0 = 63,43^\circ$

$\delta = 180^\circ - \big(\arctan(m_{AD}) - \arctan(m_{CD})\big) \approx 180^\circ - (63,43^\circ - 0) = 116,57^\circ$

Übersicht

$\overline{AB} = 3$	$m_{AB} = 0$	$M_{AB}\,(2,5\,\vert\,1)$	$\alpha \approx 63,43^\circ$
$\overline{BC} = \sqrt{5} \approx 2,24$	$m_{BC} = 2$	$M_{BC}\,(4,5\,\vert\,2)$	$\beta \approx 116,57^\circ$
$\overline{CD} = 3$	$m_{CD} = 0$	$M_{CD}\,(3,5\,\vert\,3)$	$\gamma \approx 63,43^\circ$
$\overline{AD} = \sqrt{5} \approx 2,24$	$m_{AD} = 2$	$M_{AD}\,(1,5\,\vert\,2)$	$\delta \approx 116,57^\circ$

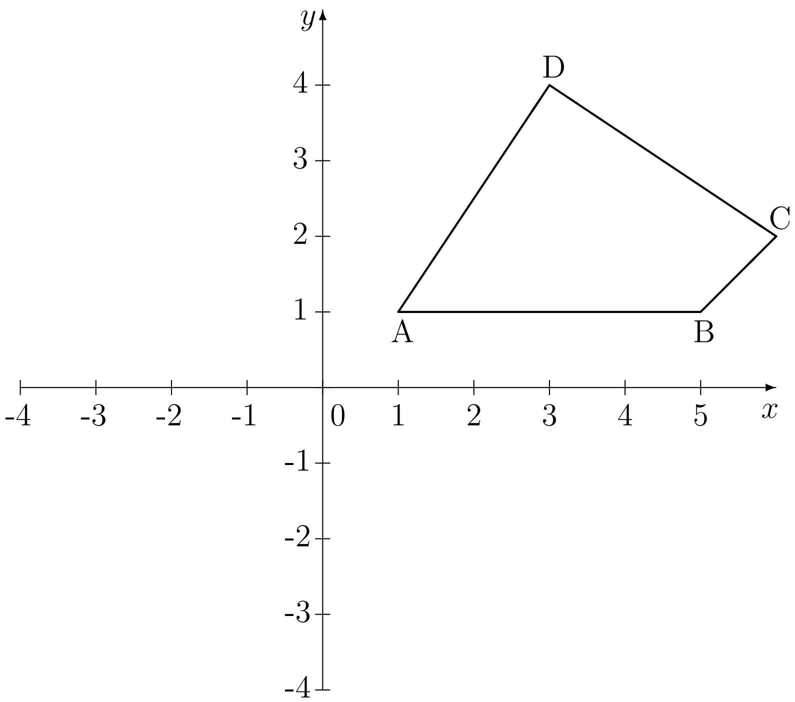
b)



Übersicht

$\overline{AB} = \sqrt{5} \approx 2,24$	$m_{AB} = 2$	$M_{AB}\,(1,5 \,\vert\, -1)$	$\alpha \approx 8,13°$
$\overline{BC} = \sqrt{18} \approx 4,24$	$m_{BC} = 1$	$M_{BC}\,(3,5 \,\vert\, 1,5)$	$\beta \approx 198,43°$
$\overline{CD} = \sqrt{5} \approx 2,24$	$m_{CD} = -\frac{1}{2}$	$M_{CD}\,(4 \,\vert\, 3,5)$	$\gamma \approx 71,57°$
$\overline{AD} = \sqrt{40} \approx 6,32$	$m_{AD} = 3$	$M_{AD}\,(2 \,\vert\, 1)$	$\delta \approx 81,87°$

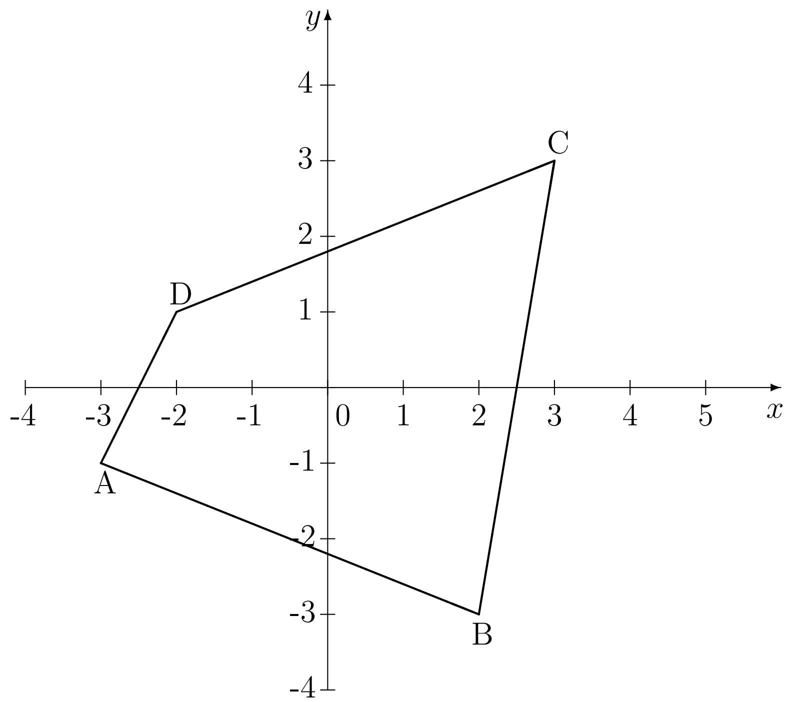
c)



Übersicht

$\overline{AB} = 4$	$m_{AB} = 0$	$M_{AB}\,(3\,\vert\,1)$	$\alpha \approx 56,31°$
$\overline{BC} = \sqrt{2} \approx 1,41$	$m_{BC} = 1$	$M_{BC}\,(5,5\,\vert\,1,5)$	$\beta = 135°$
$\overline{CD} = \sqrt{13} \approx 3,61$	$m_{CD} = -\frac{2}{3}$	$M_{CD}\,(4,5\,\vert\,3)$	$\gamma \approx 78,69°$
$\overline{AD} = \sqrt{13} \approx 3,61$	$m_{AD} = 1,5$	$M_{AD}\,(2\,\vert\,2,5)$	$\delta = 90°$

d)



Übersicht

$\overline{AB} = \sqrt{29} \approx 5,39$	$m_{AB} = -0,4$	$M_{AB}\,(-0,5\,\vert-2)$	$\alpha \approx 85,24°$
$\overline{BC} = \sqrt{37} \approx 6,08$	$m_{BC} = 6$	$M_{BC}\,(2,5\,\vert\,0)$	$\beta \approx 77,66°$
$\overline{CD} = \sqrt{29} \approx 5,39$	$m_{CD} = 0,4$	$M_{CD}\,(0,5\,\vert\,2)$	$\gamma \approx 58,74°$
$\overline{AD} = \sqrt{5} \approx 2,24$	$m_{AD} = 2$	$M_{AD}\,(-2,5\,\vert\,0)$	$\delta \approx 138,37°$